2. Mathematically compute the mean queue lengths and the server utilization and compare with the simulation results (The mathematical formulation will be discussed in class).

Server Utilization

The server utilization, the percentage of time during which the server is busy processing jobs during a simulation, can be mathematically computed by computing the average number of packets and divides by the average service time.

Arrival rate: λ

Service rate: μ

Utilization factor (proportion of time the server is busy): ρ = λ / μ

* μ = 1 packet/second, λ = 0.1

0.1 / 1 = 0.1

* μ = 1 packet/second, λ = 0.2

0.2 / 1 = 0.2

* μ = 1 packet/second, λ = 0.4

0.4 / 1 = 0.4

* μ = 1 packet/second, λ = 0.5

0.5 / 1 = 0.5

* μ = 1 packet/second, λ = 0.6

0.6 / 1 = 0.6

* μ = 1 packet/second, λ = 0.8

0.8 / 1 = 0.8

* μ = 1 packet/second, λ = 0.9

0.9 / 1 = 0.9

Mean Queue Length

From the M/N/1 Queuing System’s Results, we have further learnt

Average number of packets in the system: N = ρ / (1- ρ)

Average packet time in the system: T = N / λ = 1 / (μ - λ)

Average waiting time in queue: W = T - 1/μ = ρ / (μ - λ)

Average number of packets in queue: NQ = λW = ρ^2 / (1- ρ)

* Mean Queue Length can be computed using ρ^2 / (1- ρ)
* μ = 1 packet/second, λ = 0.1

(0.1/1)^2 / (1-(0.1/1)) = 0.01 / 0.9 = 0.01111111

* μ = 1 packet/second, λ = 0.2

(0.2/1)^2 / (1-(0.2/1)) = 0.04 / 0.8 = 0.05

* μ = 1 packet/second, λ = 0.4

(0.4/1)^2 / (1-(0.4/1)) = 0.16 / 0.6 = 0.2666666

* μ = 1 packet/second, λ = 0.5

(0.5/1)^2 / (1-(0.5/1)) = 0.25 / 0.5 = 0.5

* μ = 1 packet/second, λ = 0.6

(0.6/1)^2 / (1-(0.6/1)) = 0.36 / 0.4 = 0.9

* μ = 1 packet/second, λ = 0.8

(0.8/1)^2 / (1-(0.8/1)) = 0.64 / 0.2 = 3.2

* μ = 1 packet/second, λ = 0.9

(0.9/1)^2 / (1-(0.9/1)) = 0.81 / 0.1 = 8.1

Need to compare with the simulation result!